Statistical analysis of the extreme values of stress time series from the Portevin–Le Châtelier effect

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(Received 26 January 2004; published 21 September 2004)

In an effort to understand the deterministic vs stochastic character of the Portevin–Le Châtelier (PLC) phenomenon, we investigate the structure of the underlying mechanism that generates the stick-slip patterns of stress over time. The stress time series is reduced to a series of successive pairs of minimum and maximum values representing the stick-slip patterns and a statistical analysis by means of hypothesis testing is applied to it. The null hypothesis of least deterministic structure is that the time series of extreme values is a bounded random walk of alternating direction (BRWAD); that is, besides the constraint of succession of minima to maxima bounded at a predefined range there are no other correlations in the data. To implement the test we use surrogate data generated by a model consistent with a BRWAD type process, which also uses the statistics of the original data to best mimic them. The proposed hypothesis testing is found to perform properly on simulated data from stochastic and deterministic systems. For the PLC time series, the null hypothesis is rejected at a high level of confidence giving evidence for some deterministic structure in the succession of the extreme stress values. This result allows for further statistical analysis including also the time aspect of the stick-slip patterns.

DOI: 10.1103/PhysRevE.70.036110 PACS number(s): 02.50.Ga, 62.20.Fe, 05.45.Tp, 05.10.Ln

I. INTRODUCTION

The presence of *stick-slip* patterns (slow rather linear elasticlike up-trends followed by down-trends of fast plastic relaxation) in stress time series of metallic alloys is attributed to the *Portevin–Le Châtelier* (PLC) *effect*, a form of plastic instability [1]. The PLC effect has been the subject of extensive study. For example, models of PLC-like deformation have been presented in [2–10] and [11–13]. While these rather sophisticated, physically based mathematical models capture some of the PLC structure, they cannot explain in detail the mechanism underlying the different regimes of the PLC effect. On the other hand, stress time series obtained from constant strain rate deformation during PLC have been studied in [14–16] with techniques of nonlinear time series analysis and evidence has been reported for the presence of deterministic, nonlinear, and chaotic behavior. Still, the nature of the underlying mechanism of the PLC effect seems to need further investigation.

The succession of slow positive and fast negative linear trends of the stick-slip patterns characterizes the stress time series with data asymmetry and time irreversibility, both indicating nonlinearity and deterministic dynamics [17,18]. However, these dynamics regard small time scales and are evident even by eye-ball judgment. So, in order to get insight onto the underlying mechanism one has to investigate whether there is *long term* deterministic structure in addition to the *short term* nonlinear dynamics that forms the stick-slip patterns. In [19], a statistical test was conducted comparing stress time series of stick-slip patterns from single crystals to time series having reshuffled the stick slip patterns, but the results were not conclusive to suggest significant discrimination.

In this paper, we attempt through a statistical approach to deal with the question whether the underlying system has a long memory that spans over the time of a stick-slip pattern or the sequence of upward and downward trends of the stick-slip patterns is totally random. We simplify the data analysis by assigning one time step for the time of each upward and downward (both approximately linear) trend, i.e., we derive time series of successive maximum and minimum values from the stress time series.

The statistical analysis of the time series of extreme values is focused on the hypothesis test for randomness under the constraint of succession of minima and maxima bounded at a predefined range of values, which we call *bounded random walk of alternating direction* (BRWAD). We develop a very simple stochastic phenomenological model to generate surrogate data consistent to the null hypothesis, i.e., the surrogate time series are realizations of a BRWAD process that also mimic the original time series of extrema in terms of the amplitude distribution of the minima and maxima. This model does *not* have long term memory and generates visually indistinguishable time series from the experimental time series of extrema. The surrogate data test is applied first to simulated time series exhibiting stick-slip patterns generated by stochastic and deterministic systems to assert that it performs properly and then it is applied to some experimental stress time series.

We believe it is important to conduct a rigorous surrogate data test for the simplest hypothesis for the stick-slip patterns

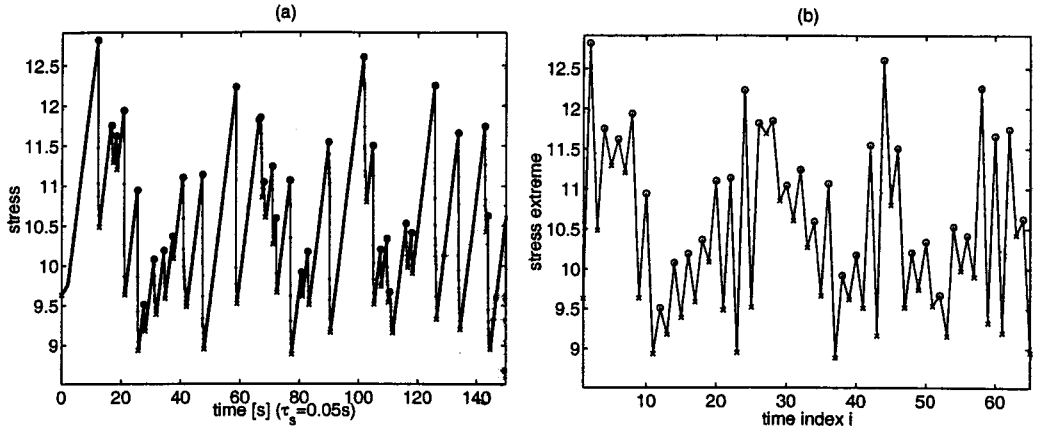

FIG. 1. (a) Segment of the stress time series S1, where the samples are denoted by connected dots, the local maxima by open circles, and the local minima by crosses. (b) The segment of the time series of extrema corresponding to the segment of the stress time series in (a). The samples of the new time series are denoted by connected open circles and crosses in alternating order and correspond to the minima and maxima in (a).

in order to assess whether long term correlations in the stress time series can be statistically established and determine the level of significance for this. The work in this paper constitutes the first approach in a step-by-step statistical analysis and phenomenological modeling of the stress time series. Thus if the working null hypothesis is rejected at high confidence level then the model of BRWAD type can be expanded to incorporate also temporal correlations.

The BRWAD model and the statistical test are presented in Sec. II. The performance of the test using BRWAD is assessed in Sec. III using simulated data and the test is applied to stress time series in Sec. IV. Discussions of the results and concluding remarks are given in Sec. V.

II. THE STATISTICAL ANALYSIS

The time series we focus our statistical analysis on are comprised of alternating extreme points, typically derived from a time series of oscillating type. Our primary interest is in time series of stick slips, such as the stress time series. In Fig. 1, a segment of the stress time series and the respective time series of extrema are shown (the stress time series in the figure is described in Sec. IV). In the time series of extrema, only the turning points of the original time series are preserved dropping all the other points, which, due to the linearity of each up and down trend, do not contain any interesting dynamical information. However, this severe filtering does not preserve the information for the time period of each trend. This kind of reduction of information is common in the analysis of time series which exhibit "exciting" variations in only a small subset of the original data set. For example, in the nonlinear analysis of time series, interspike intervals are often used instead of the complete time series [20]. Thus the time series of extrema evolves on a different time scale, i.e., a single time step in the time series of extrema in Fig. 1(b) corresponds to several time steps in the stress time series in Fig. 1(a). In this way, time undergoes a nonuniform transformation. As a consequence, the presence of one-step correlations in the time series of extrema implies the presence of long-term correlations in the stress time series.

The objective of our statistical analysis is to investigate whether it is possible that a time series of extrema, as those derived from the stress time series, be a realization of a stochastic process under the least of constraints implied by the data configuration. For this, we first build an appropriate model and then we assess the adequacy of the model using a number of statistical measures combined with the surrogate data test for the hypothesis.

A. A model for the time series of extrema

Consider the time series of extrema $x_1, x_2, \ldots, x_n$. It satisfies the constraint of consecutive minima and maxima: $x_1 < x_2$, $x_2 > x_3$, $x_3 < x_4$, etc. Furthermore, we will also make the simplifying assumption that x_1 is a minimum and x_n is a maximum. We present a probabilistic model for the generation of $x_1, x_2, \ldots, x_n$.

As a first step we introduce two auxiliary time series y_0, $y_1, y_2, \ldots, y_{n/2-1}$ and $u_1, u_2, \ldots, u_{n/2}$, defined as follows:

$$y_k = x_{2k+1} \quad \text{for} \quad k = 0,1,2,\ldots,\frac{n}{2}-1,$$

$$u_k = x_{2k} \quad \text{for} \quad k = 1,2,\ldots,\frac{n}{2},$$

i.e., we rewrite the original time series as $y_0, u_1, y_1, u_2, \ldots, y_{n/2-1}, u_{n/2}$, where the y_k's are the minima and the u_k's the maxima. For example, referring to Fig. 1(b), the first four samples are $y_0 = x_1 = 9.63$, $u_1 = x_2 = 12.82$, $y_1 = x_3 = 10.48$, and $u_2 = x_4 = 11.75$. Thus we can consider two separate time series associated to the two components appearing with period 2, one for maxima $\{u_k\}_{k=1}^{n_u}$ and one for minima $\{y_k\}_{k=0}^{n_y}$, where $n_y = n/2 - 1$, $n_u = n/2$ and $n = n_y + n_u + 1$.

We assume for the underlying process that, given y_0, the y_k's and u_k's are generated for $k = 1,2,\ldots$ by the following rule:

$$u_k = v_k(U - y_{k-1}) + y_{k-1}, \tag{1}$$

$$y_k = w_k(u_k - L) + L. \tag{2}$$

Thus the process $\{x_t\}$ is given in terms of two random processes $\{v_k\}$ and $\{w_k\}$, defined as follows: (a) for every k we have $v_k \sim V[0,1]$ and $w_k \sim W[0,1]$, where $V[0,1]$ and $W[0,1]$ are arbitrary distributions on the interval $[0,1]$ and we call them *core distributions*; (b) $\{v_k\}$ and $\{w_k\}$ are white noise, i.e., for all times i,k with $i \neq k$

$$\overline{(v_i - \overline{v_i})(v_k - \overline{v_k})} = 0,$$

$$\overline{(w_i - \overline{w_i})(w_k - \overline{w_k})} = 0,$$

$$\overline{(v_i - \overline{v_i})(w_k - \overline{w_k})} = 0 \quad \text{and} \quad \overline{(v_k - \overline{v_k})(w_k - \overline{w_k})} = 0$$

where the overbar denotes expected value.

Hence the generation of the "interleaved" y_k and u_k time series can be described as follows: the first minimum y_0 is selected randomly in the interval $[L,U]$, which forms the range for the data (actually, in the implementation we choose $y_0 \in [L,(L+U/2)]$); then at times $k=1,2,\ldots$ we select a maximum in the interval $[y_{k-1},U]$ according to Eq. (1) and a minimum in the interval $[L,u_k]$ according to Eq. (2). The process defined in this way is a type of random walk since at each iteration of the process a random move is made from the last position. The walk is bounded from above and below by the parameters U and L and at each step the direction is restricted to be opposite to the direction in the previous step. We call the model for this process *bounded random walk of alternating direction* (BRWAD). Note that the variables of this process are not identically distributed as the transform at each iteration in Eqs. (1) and (2) depends on the variable y_{k-1} or u_k. However, the upward and downward random increments (i.e., $u_k - y_{k-1}$ and $y_k - u_k$) are determined (respectively) by v_k and w_k [see Eqs. (1) and (2)], which follow the core distributions $V[0,1]$ and $W[0,1]$ and do not depend on the current position.

Note that Eq. (1) can be rewritten in the form of a random coefficients autoregressive (AR) model:

$$u_k = a_k u_{k-1} + b_k, \tag{3}$$

where the random coefficients are

$$a_k = (1 - v_k)w_{k-1} \quad \text{and} \quad b_k = (1 - v_k)(1 - w_{k-1})L + v_k U.$$

Similarly, Eq. (2) can be rewritten as

$$y_k = c_k y_{k-1} + d_k, \tag{4}$$

where

$$c_k = (1 - v_k)w_k \quad \text{and} \quad d_k = (1 - w_k)L + v_k w_k U.$$

Hence, Eqs. (3) and (4) taken together form an order one AR model with random and periodic coefficients of period 2, which is regarded as a low order nonlinear stochastic model [21].

Returning to the time series of extrema $x_1, x_2, \ldots, x_n$, this is generated by the BRWAD process in the following manner: first x_1 is chosen in the interval $[L,U]$; then (for $k=1,2,\ldots$) x_{2k} and x_{2k+1} are generated by

$$x_{2k} = v_k(U - x_{2k-1}) + x_{2k-1}, \tag{5}$$

$$x_{2k+1} = w_k(x_{2k} - L) + L. \tag{6}$$

This completes the specification of the probabilistic model of the time series of extrema.

B. Generation of surrogate data

We use the BRWAD model to generate surrogate data and test the null hypothesis that the time series does not possess correlations apart from those imposed by the succession of maxima and minima. The BRWAD model is tailored to represent the null hypothesis. The novelty of generating proper surrogate data is to match certain sample statistical properties of the original data. So, for the BRWAD model, we need to specify the bounds L, U and the core distributions $V[0,1]$ and $W[0,1]$ from the given time series of extrema $\{x_t\}_{t=1}^n$. We set the bounds to the minimum and maximum of the original time series, $L=x_{\min}$ and $U=x_{\max}$. The core distributions are formed by the empirical sample distributions estimated from $\{x_t\}_{t=1}^n$ as follows. The estimates of v_k and w_k (call them $\hat{v}_k$ and $\hat{w}_k$) can be obtained from x_t using Eqs. (5) and (6)

$$v_k = \frac{x_{2k} - x_{2k-1}}{U - x_{2k-1}} \quad \text{and} \quad w_k = \frac{x_{2k+1} - L}{x_{2k} - L}. \tag{7}$$

The sample values $\{\hat{v}_k\}_{k=1}^{n_u}$ and $\{\hat{w}_k\}_{k=1}^{n_y}$ are computed from Eq. (7) using the original data and they form the sample distributions of $V[0,1]$ and $W[0,1]$, respectively, i.e., at each iteration of the model a random component v_k and w_k is drawn with equal probability from $\{\hat{v}_k\}_{k=1}^{n_u}$ and $\{\hat{w}_k\}_{k=1}^{n_y}$, respectively.

The complete algorithm for the generation of a surrogate time series $\{z_t\}_{t=1}^n$ with BRWAD is as follows:

(1) We compute $L=x_{\min}$, $U=x_{\max}$, $\{\hat{v}_k\}_{k=1}^{n_u}$, and $\{\hat{w}_k\}_{k=1}^{n_y}$.

(2) We select $z_1=y_0$ randomly in the range $\left[L,(L+U)/2\right]$.

(3) We generate the maxima and minima of the surrogate time series as follows [recall Eqs. (5) and (6)]

$$\begin{aligned} z_{2k} &= v_k(U - z_{2k-1}) + z_{2k-1}, \quad k = 1,2,\ldots,n_u, \\ z_{2k+1} &= w_k(z_{2k} - L) + L, \quad k = 1,2,\ldots,n_y, \end{aligned} \tag{8}$$

where the components v_k and w_k are draws from $\{\hat{v}_k\}_{k=1}^{n_u}$ and $\{\hat{w}_k\}_{k=1}^{n_y}$, respectively.

C. The discriminating statistics

An important part of the statistical analysis is the estimation of linear and nonlinear characteristics of the time series of extrema. For the linear analysis, we consider the autocorrelation and the fit with a low order linear autoregressive (AR) model and for the nonlinear analysis the mutual information and the fit with a local average model. These four methods serve also as discriminating statistics for the test,

denoted in general as q, for the surrogate data test and they are briefly presented below.

1. Autocorrelation

The autocorrelation $r(\tau)$ measures the linear correlation in the time series and is defined as

$$q^{\tau}_{\mathrm{AUT}} \equiv r(\tau) = \frac{\langle (x_t - \langle x \rangle)(x_{t-\tau} - \langle x \rangle) \rangle}{\langle (x_t - \langle x \rangle)^2 \rangle}, \tag{9}$$

where $\langle x \rangle$ is the average over all available data (note that this is a time average whereas $\overline{x}$ is the expectation). The discriminating statistic of autocorrelation is computed for a range of delays τ and for each τ a separate hypothesis test is made.

2. Autoregressive fit

The fit with an AR model of order m is

$$\hat{x}_{t+1} = \phi_0 + \sum_{j=1}^{m} \phi_j x_{t-j+1}, \tag{10}$$

where the coefficients $\phi_0, \phi_1, \ldots, \phi_m$ are estimated by least-squares fit. The goodness of fit is measured here with the correlation coefficient CC(m) between true and predicted data

$$q^{m}_{\mathrm{ARF}} \equiv \mathrm{CC}(m) = \frac{\langle (x_{t+1} - \langle x \rangle)(\hat{x}_{t+1} - \langle \hat{x} \rangle) \rangle}{\sqrt{\langle (x_{t+1} - \langle x \rangle^2) \rangle \langle (\hat{x}_{t+1} - \langle \hat{x} \rangle^2) \rangle}}, \tag{11}$$

and this is the discriminating statistic for each order m.

3. Mutual information

The mutual information $I(\tau)$ measures the general correlation (linear and nonlinear) between x_t and $x_{t-\tau}$ for different delays τ and is defined as [17,22]

$$q^{\tau}_{\mathrm{MUT}} \equiv I(\tau) = \sum_{i,j} p_i \log \frac{p_{i,j}}{p_i p_j}. \tag{12}$$

In the above expression the summation is over the bins of the partition of the data (default value is 16), p_i is the estimated probability that a data point x_t is in bin i, p_j is the estimated probability that a data point $x_{t-\tau}$ is in bin j, and $p_{i,j}$ is the estimated joint probability that x_t is in bin i and $x_{t-\tau}$ is in bin j.

4. Local average mapping

For most of the methods of nonlinear time series analysis the scalars x_t are transformed to points $\mathbf{x}_t$ in $\mathbb{R}^m$ using a delay parameter τ, so that $\mathbf{x}_t = [x_t, x_{t-\tau}, \ldots, x_{t-(m-1)\tau}]'$ [17]. Here, we simply set $\tau = 1$. A local model estimates the function that maps the point $\mathbf{x}_t$ to x_{t+1} locally for each target point $\mathbf{x}_t$. We use a simple local model, called local average mapping (LAM), which predicts the one time step mapping $\hat{x}_{t+1}$ of each reconstructed point $\mathbf{x}_t$ from the average of the respective mappings of its k nearest neighbor points. The model is applied in the same way as the AR model and the discriminating statistic q^{m}_{LAM} is computed as in Eq. (11). The parameter m of LAM is called embedding dimension and has the same role as the order m for the AR model. Note that LAM is not used here as an excellent nonlinear model, but as a simple nonlinear statistic, which is actually popular in terms of the surrogate data test for nonlinearity [23,24].

All the above measures assume stationarity of the time series. The time series of extrema can be seen as nonstationary if we regard it as a concatenation of two different processes. We overlook this inconsistency bearing in mind that the estimates from the measures do not assign exact statistical properies, but they are rather used as discriminating statistics for the hypothesis test.

D. The surrogate data test

The estimation of statistical measures on a time series of extrema, as the four measures described above, may give evidence for the existence and degree of stochasticity, determinism, and nonlinearity of the underlying mechanism. For example, a moderate autocorrelation compared to a large mutual information in the first few lags may be interpreted as a sign of the existence of nonlinear determinism. Still, such evidence is incomplete if we do not know what is the range of values of the measure estimates that would be expected under the assumption of a certain system type for the data. Our interest is to investigate whether the underlying system can be regarded as purely stochastic or as one that contains some degree of determinism (or correlation) that in turn may be linear, nonlinear, or both. The use of surrogate data in hypothesis testing provides the empirical distribution of the discriminating statistic q under the null hypothesis H_0 for the nature of the underlying system. Therefore the test is considered rigorous and it can be applied also when the distribution of q is not known analytically. The empirical distribution of q is formed from the values $q_1, q_2, \ldots, q_M$ computed on an ensemble of M surrogate data consistent to H_0. So, the test decision is drawn by simply evaluating whether the statistic q_0 computed on the original data falls within the empirical distribution of q under H_0.

The working hypothesis H_0 is that the time series of extrema is generated by a system that alternates between turning points in a totally random manner, i.e., it is a process of BRWAD type. The surrogate data test is conducted in the following steps.

(1) We generate M surrogate time series $\{z^1_t\}^n_{t=1}, \{z^2_t\}^n_{t=1}, \ldots, \{z^M_t\}^n_{t=1}$, from the BRWAD model fitted to the given time series $\{x_t\}^n_{t=1}$, as described in Sec. II B.

(2) We compute one of the discriminating statistics in Sec. II C on the original data $\{x_t\}^n_{t=1}$ and on the surrogate time series $\{z^1_t\}^n_{t=1}, \{z^2_t\}^n_{t=1}, \ldots, \{z^M_t\}^n_{t=1}$ giving the estimates q_0 and $q_1, q_2, \ldots, q_M$, respectively.

(3) We reject H_0 at a significance level α (we set $\alpha = 0.05$) if q_0 lies in the tail of the distribution formed by $q_1, q_2, \ldots, q_M$, where the tail is determined by α.

The test decision in the last step can be made using the parametric or nonparametric approach.

(1) *Parametric approach*: We assume that the distribution of q under H_0 is normal (our simulations support this assumption) and we compute the so-called significance S by

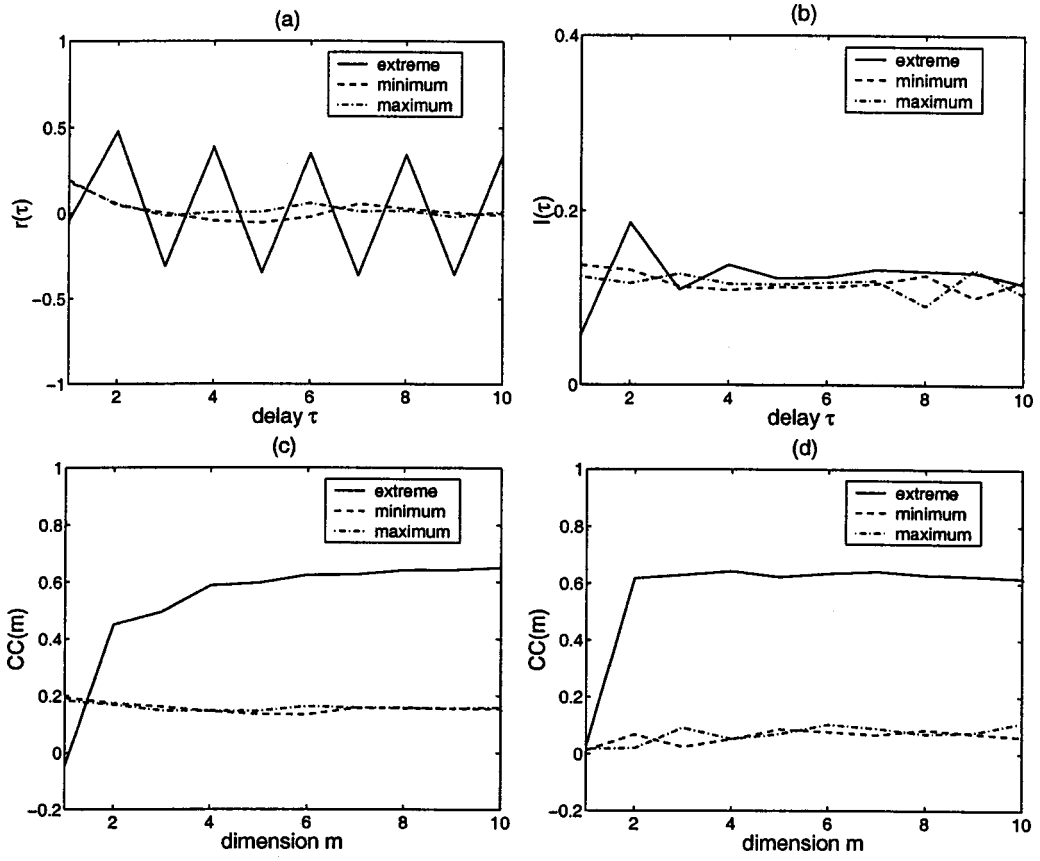

FIG. 2. Four estimates of dynamical characteristics computed on a time series of extrema, minima, and maxima, generated by the BRWAD model with uniform input noise as indicated in the legends. (a) Autocorrelation vs delay. (b) Mutual information vs delay. (c) Correlation coefficient of the fit from an AR model vs the order m. (d) Correlation coefficient of the fit from a LAM model vs the embedding dimension m.

$$S = \frac{|q_0 - \langle q \rangle|}{s_q}$$

where $\langle q \rangle$ is the average and s_q the standard deviation of $q_1, q_2, \ldots, q_M$. Significance of about 2 suggests the rejection of H_0 at the significance level $\alpha = 0.05$ (95% confidence level).

(2) *Nonparametric approach*: We order $q_0, q_1, q_2, \ldots, q_M$ and we reject H_0 if q_0 is in a position smaller than $(\alpha/2) \times (M+1)$ or greater than $(1-\alpha/2)(M+1)$ assuming a two-sided test. For $M=40$ and $\alpha=0.05$ we reject H_0 if q_0 is in the first or last position of the ordered sequence of $q_0, q_1, q_2, \ldots, q_M$.

Complementary to the surrogate data test for the time series of extrema $\{x_t\}_{t=1}^{n}$, we also perform the same test on the time series of minima $\{y_t\}_{t=1}^{n_y}$ and maxima $\{u_t\}_{t=1}^{n_u}$. The respective surrogate time series are derived from $\{z_t^1\}_{t=1}^{n}, \{z_t^2\}_{t=1}^{n}, \ldots, \{z_t^M\}_{t=1}^{n}$ accordingly.

III. PERFORMANCE OF THE TEST

We verify the validity of the statistical analysis on simulated data and study the significance and power of the surrogate data test. We chose time series of extrema from three representative systems in order to assess the consistency of the statistical analysis to the dynamical properties of the systems. The systems are a BRWAD stochastic process, a pseudoperiodic system, and a chaotic system.

A. BRWAD with uniform input noise

In Sec. II A, we designed the model BRWAD that generates stochastic time series of extrema with the least of correlations under the constraint of consecutive turning points. Here, the working data are generated by this model using standard uniform core distributions, i.e., $w_t \sim \mathrm{U}[0,1]$ and $v_t \sim \mathrm{U}[0,1]$.

In Fig. 2 we show the estimates for autocorrelation, $r(\tau)$, mutual information, $I(\tau)$, the correlation coefficient from the fit [denoted as CC(m)] with an autoregressive model AR(m), and the CC(m) from a fit with local average mapping LAM(m). The estimates are computed on a time series of extrema of $n=2048$ samples and on the respective time series of minima and maxima, where $n_y = n_u = 1024$. For the time series of minima and maxima the autocorrelation function decays exponentially to zero while for the time series of extrema it converges to a rather strong two-periodic function due to the alternating minima and maxima. This oscillating behavior of $r(\tau)$ is due to the alternation between two under-

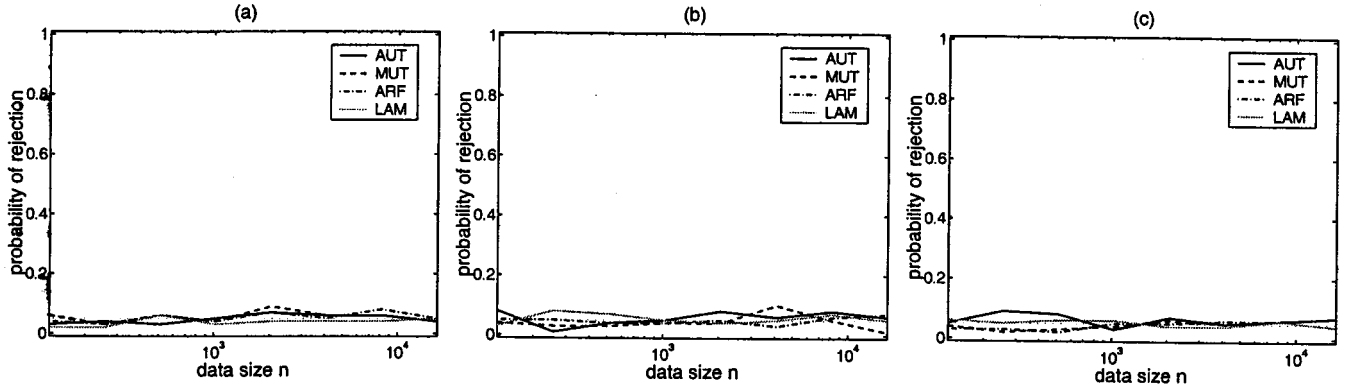

FIG. 3. The estimated probability of rejection when applying parametric surrogate data tests on 100 realizations of extrema in (a), minima in (b), and maxima in (c), generated by the BRWAD model with uniform input noise. The test results are for the statistics q^{τ}_{AUT}, q^{τ}_{MUT}, q^{m}_{ARF}, and q^{m}_{LAM}, as indicated in the legends.

lying processes which render the time series $\{x_t\}$ nonstationary. The general correlations estimated by $I(\tau)$ are also higher for the extrema than for the minima and maxima. The same feature is observed from the estimates of fit from the linear and the nonlinear model. All four measures suggest that the imposed alternations in the generation of the data result in correlated time series with the time series of extrema having distinctly strong (and linearly alternating) correlations. It seems that all correlations tend to stabilize for time windows that span over at least two samples [corresponding to $\tau>2$ for $r(\tau)$ and $I(\tau)$ and $m>2$ for AR(m) and LAM(m)].

The estimates presented in Fig. 2 are used as discriminating statistics in the surrogate data test to assess the significance (type I error) of the test, i.e., the probability of rejecting H_0 when it is true. We generated 100 time series of extrema using the BRWAD model with uniform input noise. We repeated this for a number of data sizes ranging from 128 to 16 384 with an increment of power of 2 (i.e., $7,\ldots,14$). For each one of the 100 realizations, $M=40$ surrogate time series were generated using the BRWAD model as described in Sec. II A. The discriminating statistics were computed on the original and surrogate data varying the free parameter of each measure in the same way as we did for Fig. 2, i.e., lag $\tau=1,\ldots,10$, for the statistic of autocorrelation q^{τ}_{AUT} and the statistic of mutual information q^{τ}_{MUT}; order (or embedding dimension) $m=1,\ldots,10$ for the statistic of the correlation coefficient of the fit from AR and LAM, q^{m}_{ARF} and q^{m}_{LAM}, respectively. Then we estimated the probability of rejection (counting the percentage of rejections out of 100 realizations) at the significance level of $\alpha=0.05$ for each test. The total number of tests for each of the 100 realizations is the product of the following factors:

(i) three types of time series (extrema, minima, and maxima);

(ii) seven data sizes $(2^7,2^8,\ldots,2^{14})$;

(iii) four discriminating statistics (q^{τ}_{AUT}, q^{τ}_{MUT}, q^{m}_{ARF}, and q^{m}_{LAM}); and

(iv) ten values of the free parameter (τ or m, from 1 to 10).

The results showed excellent robustness for all different factors as the probability of rejection was always at the significance level (for $\alpha=0.05$ we found about five rejections in 100 tests). For the time window of two ($\tau=2$ or $m=2$ depending on the statistic) we show in Fig. 3 the results of the probability of rejection for the range of data sizes. It is noted that the nominal probability ($\alpha=0.05$) was obtained even for realizations of 128 extrema and 64 minima and maxima and for all four statistics. In Fig. 3 the results are obtained using the parametric approach. The nonparametric approach gave qualitatively the same results.

B. Pseudoperiodic system

The pseudoperiodic systems are nonlinear deterministic systems which have nontrivial dynamics and maintain some degree of irregularity. In the simulations, we use a 2-torus in a fourth-dimensional space described in [25]. The time series is derived as the sum of the second and fourth system variables giving similar stick-slip patterns to those observed in PLC. The sampling time is $\tau_s=0.1$s and the distribution of the periods of the oscillations (of stick-slip type) has a peak at 20 samples.

Obviously, pseudoperiodic systems cannot be modeled by stochastic systems and therefore the BRWAD model should fail when applied to the time series of extrema derived by such a system. Our simulations showed that the time series of extrema, minima, and maxima from the pseudoperiodic system is discriminated from BRWAD surrogates even when the time series are small and noisy. In particular, we assess the power of the four statistics of the surrogate data test on small time series, noise-free and corrupted with up to 60% observational noise (meaning that we added white normal noise with standard deviation being 60% of the standard deviation of the data). The results are shown in Fig. 4. The simulation setup is as for the BRWAD model above.

The power of the measures decrease with the increase of noise amplitude. For example, as Fig. 4(a) shows, while the power of q^{τ}_{MUT} for noise-free data is 1 for all τ, when the data are corrupted with 60% noise its power drops to about 0.05 for all τ. The statistic q^{τ}_{AUT} seems to be the most robust to noise, but has generally smaller and varying power with the free parameter, as compared to the other three statistics. The statistics q^{m}_{ARF} and q^{m}_{LAM} reach the highest level of power in

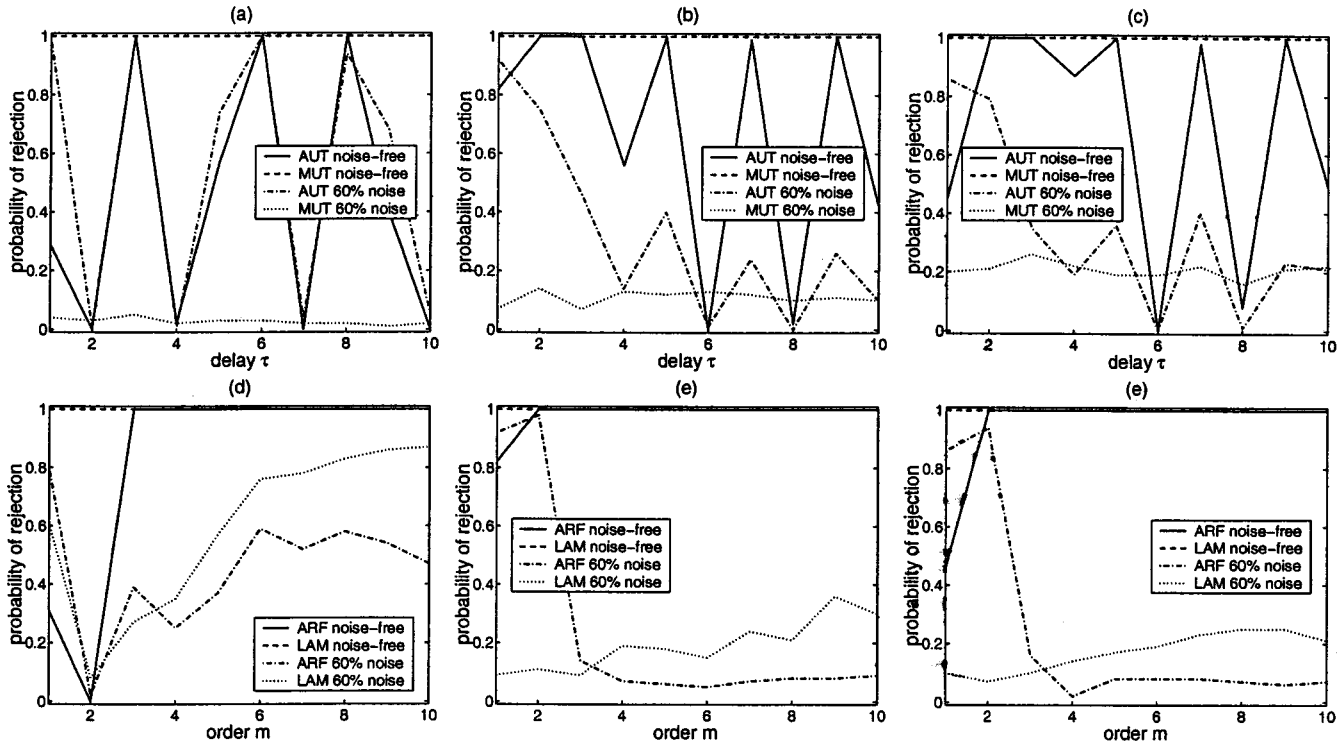

FIG. 4. The estimated probability of rejection from 100 parametric surrogate data tests for the pseudoperiodic system. The data length of the time series of extrema is $n=128$. The statistics q^{τ}_{AUT} and q^{τ}_{MUT} are shown in the panels (a), (b), and (c) for the extrema, minima, and maxima, respectively. The statistics q^{m}_{ARF} and q^{m}_{LAM} are shown in the panels (d), (e), and (f) for the extrema, minima, and maxima, respectively. The results are for noise-free time series and time series with 60% white observational noise as denoted in the labels.

the noise-free case, but their power decreases in various ways when the data are corrupted with high degree of noise and less for the whole time series of extrema than for the time series of minima and maxima. This is somehow expected as in the presence of high levels of noise the deterministic structure of the pseudoperiodic time series is masked and the original time series cannot be clearly distinguished from the BRWAD counterparts.

C. Chaotic system

We consider here the extreme time series from the fourth variable of the system of Rössler hyperchaos, which is a fourth order differential deterministic system that can exhibit stochastic behavior [26]. The oscillations of this time series are of the stick-slip type. The sampling time is $\tau_s=0.1$ s and the period of stick-slips has a rather spread distribution with a mean at about 12 samples. Besides its randomlike behavior, the system has nontrivial long term correlations that span over a single stick-slip, i.e., over many samples in the time series of extrema. However, in order to identify these correlations longer time series than the ones from the pseudoperiodic system are required.

Our simulations confirmed the dependence of the power of the statistics of the surrogate data test on the data size. In Fig. 5, results are shown from the simulations with time series lengths of extrema of $n=128$ and $n=1024$. Obviously, the power of all four statistics increase with the data size. The statistic q^{τ}_{MUT} has very small power when $n=128$ and has generally the worst performance. The other three statistics seem to have about the same power for small n, but for large n, q^{m}_{ARF} and q^{m}_{LAM} reach the highest power (for $m\geqslant 2$), with q^{m}_{LAM} performing best.

In general, the surrogate data test seems to work properly with all four statistics, giving small significance when the original time series is consistent to H_0 and large power when the original time series is not consistent to H_0. The power depends on the data size and the noise level. One cannot assign more specific rules for the power of the test as it is heavily system dependent.

IV. APPLICATION TO STRESS TIME SERIES

We use the time series of total stress from two experiments exhibiting the PLC effect (the time series are the same as in [27]). The first experiment is on a single crystal Cu-10% Al compressed at constant strain rate $\dot{\epsilon}=3.3\times10^{-6}\ \mathrm{s}^{-1}$. The stress is sampled at a sampling time $\tau_s=0.05$ s during stage I (Lüders deformation) with zero average hardening. So, the selected stress time series of 20 000 samples is regarded stationary and therefore no detrending was applied. The stress time series is comprised of stick-slip patterns, which have a distinctly linear and slow up-trend followed by a very rapid down-trend. The duration of the stick-slip patterns has a spread distribution with an average of about 100 samples. The peaks and troughs of the stick-slips are clearly discernible, which accommodated the computation of the local extrema (see Fig. 1). The extracted time

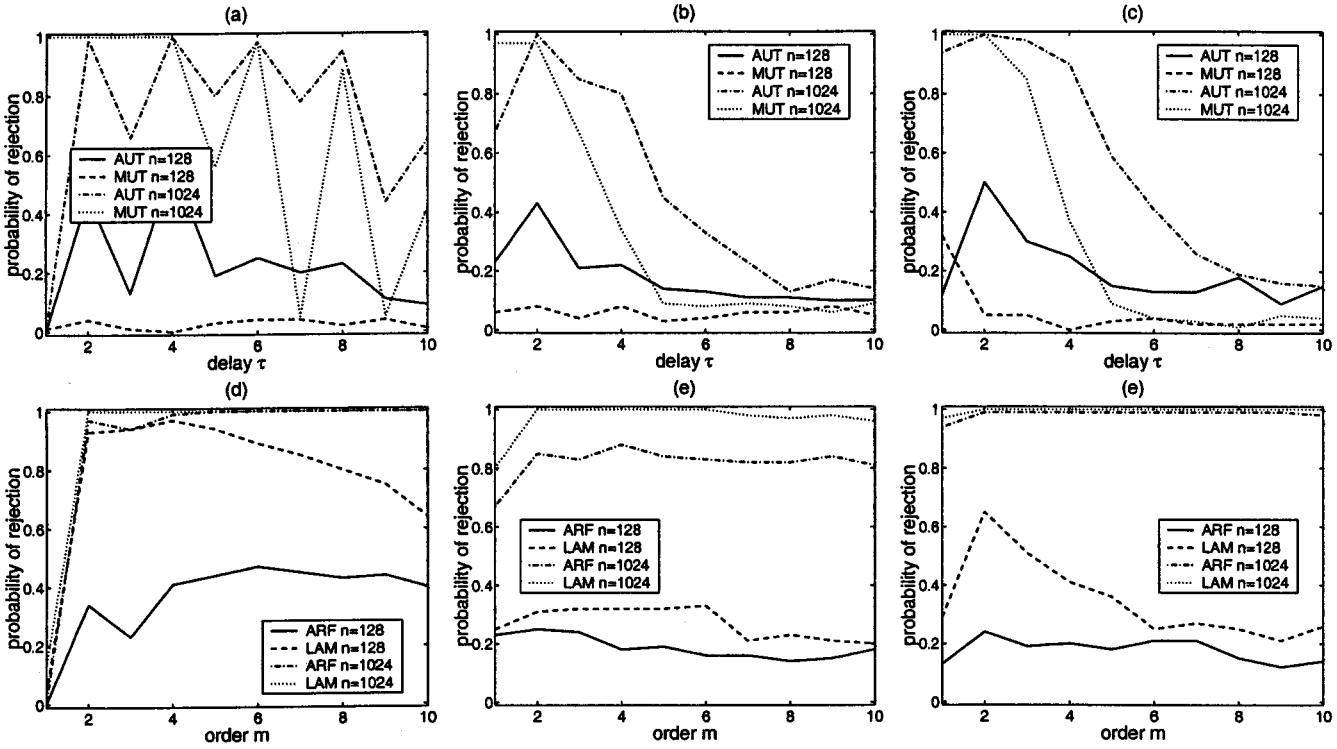

FIG. 5. The estimated probability of rejection from 100 parametric surrogate data tests for the Rössler hyperchaos system. The statistics q^{τ}_{AUT} and q^{τ}_{MUT} are shown in the panels (a), (b), and (c) for the extrema, minima, and maxima, respectively. The statistics q^{m}_{ARF} and q^{m}_{LAM} are shown in the panels (d), (e) and (f) for the extrema, minima, and maxima, respectively. The results yield two lengths of the time series of extrema, $n=128$ and $n=1204$ as denoted in the labels.

series of extrema has length $n=358$ and it is denoted S1.

We also use three time series from a polycrystal Cu-15% Al strained under tension at $\dot{\epsilon}=100\times 6.67\times 10^{-6}\ \mathrm{s}^{-1}$ and $T=250\,^{\circ}\mathrm{C}$, sampled at $\tau_s=0.02\mathrm{s}$. Three successive segments of length 3036 samples each were obtained. An increase in stress due to work hardening could be seen as a small trend in the segments that was removed using a 5-deg polynomial. The mean period of stick-slips was about eight samples in all three segments. The respective time series of extrema were derived in the same way as for the single-crystal and the notations and lengths of the three time series are P1 and $n=816$, P2 and $n=870$, and P3 and $n=848$.

We apply the test with the BRWAD surrogates to the four time series of extrema S1, P1, P2, and P3. The general result from the surrogate data test on the time series of extrema is that P1, P2, and P3 are clearly discriminated by the respective BRWAD surrogates while for S1 significant discrimination is attained only for certain values of the parameters of the discriminating statistics. Note that S1 is from an experiment with single-crystal and its length is less than half of the lengths of the other three time series, which are derived from the experiment on polycrystal alloy. Recalling the results on the simulated data the limited length of S1 might be a possible reason for the less significant discrimination.

The test was done on the whole time series of extrema and on the time series of minima and maxima, separately. It turned out that in all cases the discrimination between original and BRWAD surrogates was less significant for the time series of minima and maxima than for the whole time series. As shown in Fig. 6 for the statistics q^{τ}_{AUT} and q^{τ}_{MUT} (for $\tau=1,\ldots,10$), using the parametric approach the significance S for the surrogate data test is consistently larger for the time series of extrema than for the time series of minima and maxima. Note that H_0 is rejected at 95% confidence level when $S>1.96$ and this threshold of S is shown with a gray line in the panels of Fig. 6. The statistic q^{τ}_{AUT} seems to have larger discriminating power than q^{τ}_{MUT}. For example, for the S1 time series of extrema, q^{τ}_{AUT} gives $S>2$ for even τ while q^{τ}_{MUT} gives only marginal rejection of H_0 for $\tau=2$ and $\tau=4$ and no rejection for the other lags [see top panels of Figs. 6(a) and 6(b)]. Also, for the time series P1, P2, and P3 of minima, q^{τ}_{AUT} gives $S>2$ for $\tau<4$ while q^{τ}_{MUT} gives only sporadic rejections (at $\tau=5$ for P1 and at $\tau=1$ for P3) [see middle panels of Figs. 6(a) and 6(b)]. The same holds also for the time series of maxima, but with somewhat smaller significance.

The statistics q^{m}_{ARF} and q^{m}_{LAM} confirmed that S1 is more consistent with the BRWAD process than the other three stress time series of extrema, as shown in Fig. 7. With regard to S1, $S>2$ was obtained only for the time series of extrema at $m<6$ with q^{m}_{ARF} and at $m=1$ with q^{m}_{LAM}. In the case of extrema, for P1, P2, and P3 confident rejections were obtained from both q^{m}_{ARF} and q^{m}_{LAM} for the whole range of m. For the minima and maxima, the significance was lower and only P1 and P3 could be clearly discriminated by both methods [see middle and lower panels of Figs. 7(a) and 7(b)]. These two statistics seem to perform similarly and they seem to give more significant rejections than q^{τ}_{AUT} and q^{τ}_{MUT} and for a larger range of the free parameter.

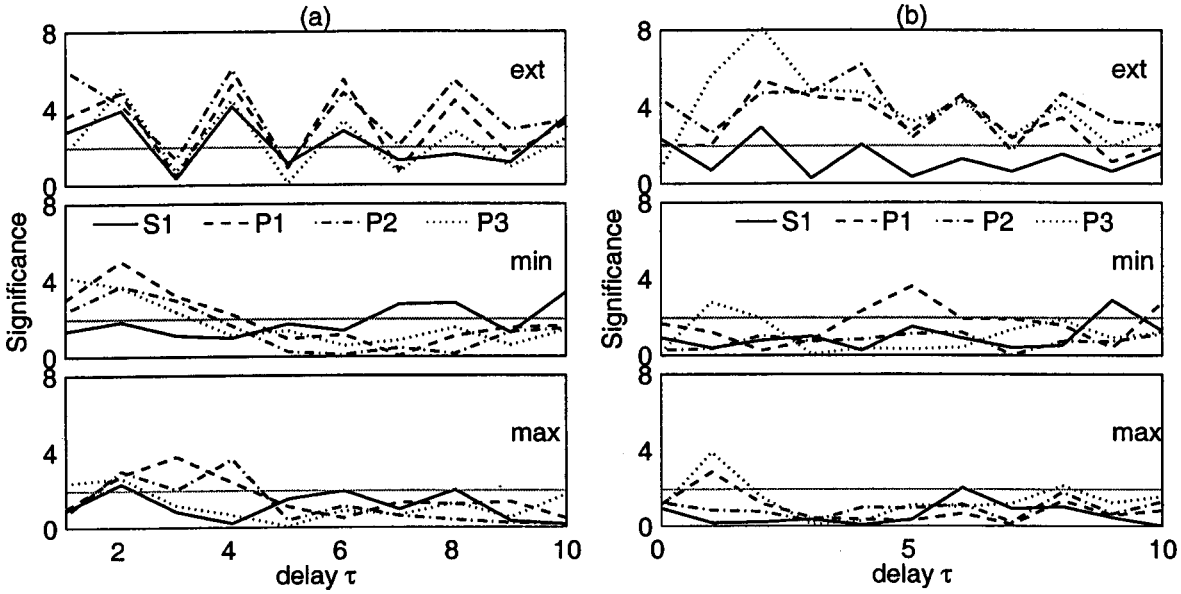

FIG. 6. (a) Significance vs τ from the test with the statistic q^{τ}_{AUT} on the four stress time series of extrema with the line types as shown in the inset. The top panel is for the whole time series of local extrema, the middle panel for the minima, and the lower panel for the maxima. (b) The same as for (a) but for the statistic q^{τ}_{MUT}. The gray horizontal line displays the threshold of rejection at the $\alpha=0.05$ significance level.

The significance is generally larger for the time series of extrema than for the time series of minima and maxima. The overall results suggest that the time series of extrema from P1, P2, and P3 are not generated by a BRWAD process and therefore we can conlude that these stress time series have nontrivial correlations between successive stick-slip patterns. For the S1 time series of extrema, the test did not give conclusive results as the hypothesis of a BRWAD generating process could be rejected only with some measures and for few values of the free parameter. This result on the single crystal is in agreement with another statistical analysis indicating also that the long range correlations in stress data are weak [19].

V. CONCLUSION

We investigated the deterministic vs stochastic character of the PLC serrations. We concentrated on long term correlations and therefore we considered the time series of extrema comprised of the turning points of the original time series in the order of appearance. For this time series we created a model of bounded random walk of alternating di-

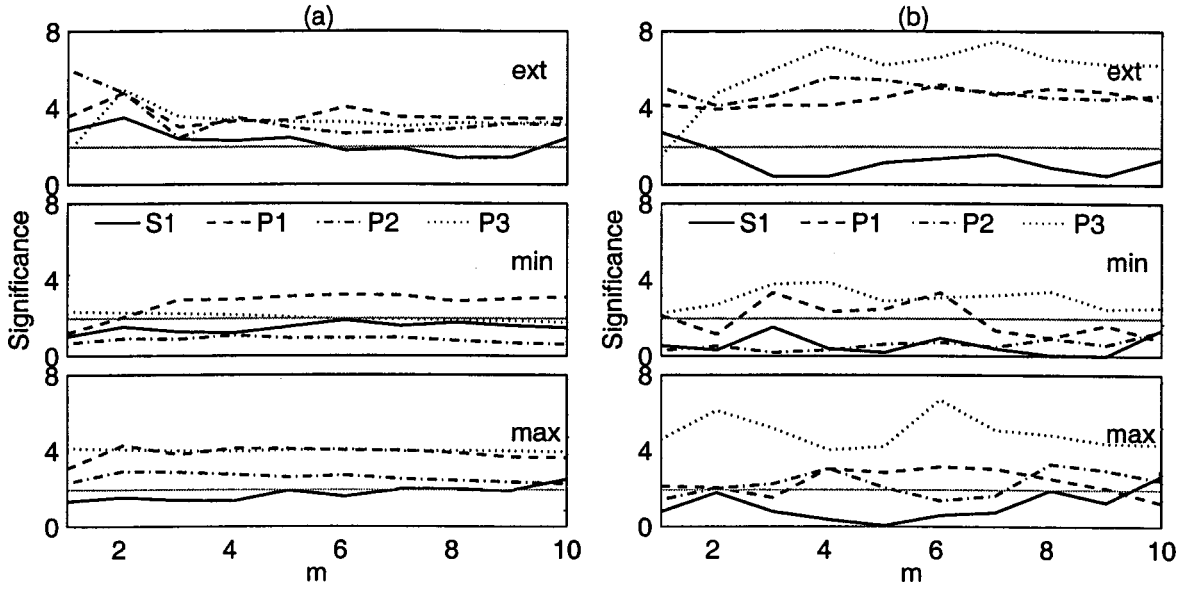

FIG. 7. (a) Significance vs m from the test with the statistic q^{m}_{ARF} on the four stress time series of extrema with the line types as shown in the inset. The top panel is for the whole time series of local extrema, the middle panel for the minima, and the lower panel for the maxima. (b) The same as for (a) but for the statistic q^{m}_{LAM}. The gray horizontal line displays the threshold of rejection at the $\alpha=0.05$ significance level.

rection (BRWAD) that assumes the least structure and generates random data under the constraint of alternating direction at each iteration. Such a model has the smallest possible memory as the only correlations in the data are formed from the alternation of random turning points.

We designed a surrogate data test for the null hypothesis (H_0) that the time series of extrema is generated by a BRWAD process. We considered four statistics for the test, the autocorrelation q^{τ}_{AUT}, the mutual information q^{τ}_{MUT}, the fit with an autoregressive model q^{m}_{ARF} and the fit with a local average map q^{m}_{LAM}. The simulated results suggest that all four statistics give small significance to the test and have varying power according to the data size and noise in the data, with q^{τ}_{MUT} having the least power for small or noisy time series.

We applied the surrogate data test to four stress time series, one obtained from a single crystal and three obtained from polycrystals and found that all time series of extrema were not consistent with the BRWAD process. For the single crystal time series in particular, the discrimination was much less significant. It is notable that when the test was applied to the separate time series of minima and maxima, the discrimination from the surrogate data was in general substantially smaller, so that in many cases rejection of H_0 could not be achieved.

The general conclusion is that the simple, short memory model does not fully explain the observed behavior of the experimental time series. The rejection of H_0 supports the assumption for the presence of deterministic structure and long term memory in the sequence of stick-slip events. More specifically, our *Markovian model* with one-step memory in the reduced time scale cannot adequately explain the observations. In other words, it appears that the sequence of stick-slip events possesses longer memory that spans over several stick-slip events and hence the system can be considered to have long-term memory.

Regarding the experimental stress data, it is possible to give a physical interpretation for the existence of long-term memory. The microstructure of the specimens changes with increasing deformation by the refinement of the dislocation cell structure (substructure) inside the grains [28–32]. The main part of the flow stress increases due to this refinement, indicating that the amplitude of the internal stress fluctuations also increases (up to some limit at elevated degree of deformation), in addition to their decrease of wavelength. Although in this work the overall flow stress increase has been removed by subtracting from the raw data a function fitted to the average flow stress, it is reasonable to assume that the change of microstructure still appears in the long-term memory discussed above. It is also well known [31] that, due to the activation of several slip systems in each grain, the fluctuations in polycrystals tend to be smoothed out to a larger extent than in single crystals, which also fits well to the result found from the above time series. In the single crystal case, there is no memory for the band because it moves during Lüders straining into virgin material, while for the polycrystal case the previous work hardened state is reflected as some memory during propagation of the next band [33].

The findings of this work open two possible directions for further statistical analysis on the PLC data. First, the results on PLC data give evidence against our hypothesis that the core processes $\{v_k\}$ and $\{w_k\}$ are white. A natural refinement of our model could be to create colored noise processes $\{v_k\}$ and $\{w_k\}$, possessing the empirically observed autocorrelation. Hence the same setup of hypothesis testing for the PLC time series could be made using a "correlated BRWAD" model in order to investigate whether this model can reproduce properties of the PLC time series of extrema.

Second, in this work the information regarding the time scale of the original time series was suppressed. A possible extension is to postulate a *vector* process which describes both the extrema and the time increments between consecutive extrema. In this way, the original time series is reduced to a new time series of the form $(\tau_1,x_1),(\tau_2,x_2),\ldots,(\tau_n,x_n)$, where (τ_i,x_i) are the coordinates for the ith extreme. In this connection, it is pointed out that for the PLC time series with linear up and down trends, the time series $\{(\tau_i,x_i)\}$ retains the most relevant information about the original time series. Thus it would be interesting to investigate whether an extended (white or correlated) BRWAD model for the vector time series $\{(\tau_i,x_i)\}$ is adequate.

ACKNOWLEDGMENTS

This research was supported in part by the European Commission under TMR ERB FMRX-CT96-0062 and RTN-CT-2002-00198 research training networks. The authors want to thank Dr. C. Engelke, who kindly provided the experimental stress time series used in the analysis.

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